[7] Slattery, R. E. and Clay, W. G., "Laminar-turbulent transition and subsequent motion behind hypervelocity spheres," ARS J. **32**, 1427–1429 (1962).
[8] Demetriades, A., "Some hot-wire anemometer measurements in a hypersonic wake," *Proceedings of the 1961 Heat Transfer and Fluid Mechanics Institute* (Stanford University Press, Stanford, Calif., 1961).
[9] Gold, H., "Stability of laminar wakes," Ph.D. Thesis, California Institute of Technology, Pasadena, Calif. (1963).
[10] Kronauer, R. E., "Growth of regular disturbances in axisymmetric laminar and turbulent wakes," Avco RAD TM-64-3 (February 17, 1964).

Reply by Author to J. I. Erdos and H. Gold

SEYMOUR L. ZEIBERG*
General Applied Science Laboratories, Inc., Westbury, N. Y.

ERDOS and Gold, in the preceding comment, question the value of a simplified correlation of wake transition data[1]; however, the fact remains that the available ballistic range data are inconclusive with regard to model size dependence,[1,2–6] and small differences in body shape.[1,2,5] Thus, an involved correlation scheme based upon these data does not seem to be indicated at present. The behavior shown schematically in Fig. 1 of the foregoing comment is not demonstrated by the data (with the possible exception of a hint at the shape by two points of the sphere data[8,7] at $M_\infty \simeq 7.5$, i.e., region 7 on Fig. 2 of the preceding note) even though the trend may be implied by the initial results of stability investigations. Therefore, when using the data outside of the range of the experiments, one must be careful to avoid reading too much from the data; e.g., note the reversals in the effect of body size over the Reynolds number range for the cases shown in Fig. 3 of the preceding note (also see Fig. 1).

With regard to the comparison of correlations shown in Fig. 3 of the preceding comment, the author notes that, according to the curve-fit described in Ref. 9, the extrapolation of the "unified transition correlation" (Fig. 4 of Ref. 1 or Fig. 2 of Erdos and Gold's note) to $M_\infty = 22$ should indicate $(Re_{x_{TR}})^2_\infty\ (M_\infty/M_e) = 10^8$. For a 12° cone at $M_\infty = 22$, the quantity M_e (as defined by the author in Ref. 1) is about 14.5; then $(Re_{x_{TR}})_\infty \simeq 4.3(10)^7$, and the comparison of the author's correlation and that of Ref. 2 appears as shown in Fig. 1. It is seen that, for the larger bodies, agreement to within a factor of 3 (which is within the experimental data scatter according to all available correlations) is obtained for altitudes of 200 kft and below.

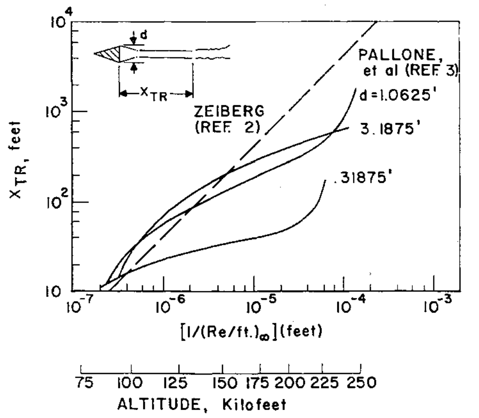

Fig. 1 Wake transition predictions for 12° cone at 22,000 fps.

Received June 15, 1964.
* Project Scientist. Member AIAA.

References

[1] Zeiberg, S. L., "Transition correlations for hypersonic wakes," AIAA J. **2**, 564–565 (1964).
[2] Pallone, A. J., Erdos, J. I., and Eckerman, J., "Hypersonic laminar wakes and transition studies," AIAA J. **2**, 855–863 (1964).
[3] Lees, L., "Hypersonic wakes and trails," AIAA J. **2**, 417–428 (1964).
[4] Webb, W. H., Hromas, L., and Lees, L., "Hypersonic wake transition," AIAA J. **1**, 719–721 (1963).
[5] Levensteins, Z., "Hypersonic wake characteristics behind spheres and cones," AIAA J. **1**, 2848–2850 (1963).
[6] Smith, C. E., "Correlation of hypersonic wake transition data," Lockheed Missile and Space Co., RN 264-22 (March 1964).
[7] Slattery, R. E. and Clay, W. G., "The turbulent wake of hypersonic bodies," ARS Preprint 2673-62 (November 1962).
[8] Slattery, R. and Clay, W., "Laminar-turbulent transition and subsequent motion behind hypervelocity spheres," ARS J. **32**, 1427–1429 (1962).
[9] Zeiberg, S. L., "Correlation of hypersonic wake transition data," General Applied Science Labs., TR 382 (October 1963).

Comment on "Derivation of Element Stiffness Matrices"

BRUCE IRONS* AND KEITH DRAPER†
Rolls-Royce Limited, Derby, England

REFERENCES 1 and 2 describe methods of structural analysis in which the deflections of an element are expressed in terms of a number $N = n + l$ basic displacement functions. The element is loaded and/or attached to other elements at its nodes, which have n slopes and deflections. The difference between this and previous work is that l, the number of surplus shape functions, was previously assumed zero.

A comparison of Refs. 1 and 2 is of interest in that the solutions are independent, are expressed differently, and have different motivation. Pian remarks that by taking a large number l of surplus undetermined coefficients α, the equilibrium conditions are improved. Reference 1 suggests that the first few α, say $\alpha_1 \ldots \alpha_r$, may represent the necessary rigid body motions; this guarantees exact equilibrium. Experience with large l in a two-dimensional problem indicates that the difficulties of imposing conformity of slopes and deflections between elements increase with l.

In the $[G]$ of Ref. 2, the first r rows and columns will be zero because the rigid body motions do not contribute to the strain energy. Some arithmetic may then be saved by using only the nonzero terms of $[G]$, say $[G_0]$, a $(n - r) \times (n - r)$ matrix. Because $\alpha_1 \ldots \alpha_r$ are of no subsequent interest, the first r rows of $[B_a{}^{-1}]$ may be discarded leaving $[C_0]$, say, a $(n - r) \times l$ matrix. Then (8) of Ref. 2 becomes

$$\{\alpha^*\} = \begin{Bmatrix} \alpha^* \\ \alpha_b \end{Bmatrix} = \left[\begin{array}{c|c} \underset{(n-r)\times n}{C_0} & \underset{(n-r)\times l}{-C_0B_b} \\ \hline \underset{l \times n}{0} & \underset{l\times l}{I} \end{array}\right] \begin{Bmatrix} q \\ \alpha_b \end{Bmatrix} = [M^*]\begin{Bmatrix} q \\ \alpha_b \end{Bmatrix}$$

where the asterisk means that the first r rows are missing.

Received May 5, 1964.
* Senior Stress Engineer, Aero Engine Division.
† Stress Engineer, Aero Engine Division.

The partitioned solution of Ref. 2 uses smaller matrices than the $[Z^{-1}]$ technique of Ref. 1. However, it becomes necessary to choose in advance which of the α are to be surplus and which can safely be put in (1, 1) correspondence with the nodal deflections. This problem does not arise with the $[Z^{-1}]$ technique.

Arithmetically it is not yet clear which is the better. Methods of inversion are known which take full advantage of symmetry, using just over $\frac{1}{2}$ N^2 words of storage and $\frac{1}{2}$ N^3 multiplications to invert an $N \times N$ matrix. A modified Waugh and Dwyer technique achieves this, as does the row-inversion technique discussed in Ref. 3. These techniques are normally recommended only for positive definite matrices. It is the authors' feeling that such a technique is applicable with a modified order of pivoting. If such a technique is applicable, the partitioned solution will use only slightly fewer operations than the $[Z^{-1}]$ solution. Nothing is yet known concerning the relative numerical accuracy.

References

[1] Irons, B. and Barlow, J., "Comment on 'Matrices for the direct stiffness method,' " AIAA J. 2, 403–404 (1964).

[2] Pian, T. H. H., "Derivation of element stiffness matrices," AIAA J. 2, 576–577 (1964).

[3] Asplund, S. O., *Structural Mechanics* (Chalmers Tekniska Hogskola, Gothenburg, 1963), Vol. II, Sec. Mg.

Comments on "Sputtering in the Upper Atmosphere"

R. V. Stuart*
Litton Industries, Minneapolis, Minn.

WEIGHT losses from a gold surface orbiting at 200 km are found by McKeown et al.,[1] the angle between the surface and the direction of motion being 30°. The authors ascribe these weight losses to sputtering of gold by N_2, the impact energy being 9 ev, and calculate an erosion rate of 0.1 Å/day or about 5×10^8 Au atoms/sec-cm². In view of results of sputtering-yield studies at very low bombarding ion energies,[2] it seems doubtful that any detectable sputtering would occur at 9 ev. It seems more likely that the weight losses observed are caused by outgassing. For N_2 outgassing, the observed weight losses would correspond to an outgassing rate of 10^{-10} torr-liters/sec-cm², which is very low for an unbaked surface.

Aside from this, since the authors indicate that they consider their results to be an upper limit on sputtering from a satellite surface, it is well to point out an apparent arithmetical error. The sputtering yield may be calculated from

$$Y = n/Nv \sin\theta$$

where

n = 5×10^8 Au atoms/sec-cm²
N = 7.82×10^9 N_2/cm³ (Ref. 3)
v = impact velocity = 8×10^5 cm/sec
θ = 30°

These data give $Y = 1.6 \times 10^{-7}$ Au/N_2, essentially an order of magnitude lower than the value given by McKeown et al.

References

[1] McKeown, D., Fox, M. G., Schmidt, J. J., and Hopper, D., "Sputtering in the upper atmosphere," AIAA J. 2, 400–401 (1964).

[2] Stuart, R. V. and Wehner, G. K., "Sputtering yields at very low bombarding ion energies," J. Appl. Phys. **33**, 2345–2352 (1962).

[3] *U. S. Standard Atmosphere* (U. S. Government Printing Office, Washington, D. C., 1962), p. 81. We follow the assumption by McKeown et al. that the composition in the upper atmosphere is mainly N_2.

Received April 20, 1964.

* Principal Scientist, Applied Science Division.

Reply by Authors to R. V. Stuart

D. McKeown* and M. G. Fox†
General Dynamics/Astronautics, San Diego, Calif.
J. J. Schmidt‡
Air Force Cambridge Research Laboratories, Sunnyvale, Calif.
and
D. Hopper§
Lockheed Missiles and Space Company, Sunnyvale, Calif.

STUART'S comment on outgassing has been previously considered in the referenced papers[1,2] in which it was concluded that the measured weight loss was due to sputtering. The weight loss was attributed to sputtering and not to out-gassing for the following reasons. First, in making an erosion measurement, not one but four gold surfaces were exposed to the vacuum of space. The gold surfaces were plated on two matched quartz oscillator crystals. Only one of the plated surfaces was actually under molecular bombardment. A photograph of the erosion gage is shown in one of the referenced articles.[2] The output of the gage is the beat frequency of the two crystals. Any outgassing should produce equal mass losses from all four of the gold surfaces, and the frequency of both crystals will increase. This frequency increase is cancelled at the gage output since only the beat frequency is telemetered. The mass change sensed by the gage will be that produced by particles impacting on the surface exposed to the molecular stream.

To check this assumption, a control gage with gold-plated crystals as well as a test gage with gold-plated crystals were flown on Discoverer 26.[1] Mass measurements were not taken until the satellite was in orbit for four days to permit the crystals to outgas. The output of the two gages was monitored closely for the following two days. The results of the test were referenced.[2] The maximum error in measuring the thickness of surface eroded that could be attributed to variation in the power supply voltage, temperature changes, outgassing, and any other unknown cause was found to be ±0.05 Å/day. The erosion rate reported in Ref. 4 was given then as 0.1 ± 0.05 Å/day, and it was reasonable to assume it was caused by sputtering from the results of the previous work.

In calculation of the sputtering yield an apparent error is present if one considers that a satellite has a circular orbit. Satellites do not. Since the atmospheric density drops off rapidly with altitude (between 200 and 330 km, the atmospheric density decreases by an order of magnitude), an average density must be used in calculating the yield. As a result of this density variation, a satellite generates a molecular beam that is intensity modulated as it passes between perigee and apogee.

Received May 25, 1964.

* Senior Staff Scientist, Space Science Laboratory. Associate Fellow Member AIAA.
† Development Engineer, Space Science Laboratory.
‡ Staff Scientist, Resident Scientist Office.
§ Flight Test Analytical Engineer.